

Mechanics M1
Advanced/Advanced Subsidiary

| Wednesday 8 June 2016 - Morning | Paper Reference |
| :--- | :--- |
| Time: $\mathbf{1}$ hour $\mathbf{3 0}$ minutes | WME01/01 |

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You must have:
Mathematical Formulae and Statistical Tables (Blue)
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Total Marks

> Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

## Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided - there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of $g$ is required, take $g=9.8 \mathrm{~m} \mathrm{~s}^{-2}$, and give your answer to either two significant figures or three significant figures.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.


## Information

- The total mark for this paper is 75 .
- The marks for each question are shown in brackets - use this as a guide as to how much time to spend on each question.


## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.


1. A car is moving along a straight horizontal road with constant acceleration $a \mathrm{~m} \mathrm{~s}^{-2}(a>0)$. At time $t=0$ the car passes the point $P$ moving with speed $u \mathrm{~m} \mathrm{~s}^{-1}$. In the next 4 s , the car travels 76 m and then in the following 6 s it travels a further 219 m .

Find
(i) the value of $u$,
(ii) the value of $a$.


Frost Pat of the owner
Entire journey
$s=76 m$ ti ss $u=, n \quad a=7 ?$
$S=29 \mathrm{~S}_{m}+\operatorname{los} U=? ? \quad a=$ ??
$S=u t+\frac{1}{2} a t^{2}$
$S=U t+\frac{1}{2} a t^{2}$
$76=4(4)+\frac{1}{2} n(4)^{2}$
$295=u(10)+\frac{1}{2} a(10)^{2}$
$76=4 u+8 a \quad(-4)$
$29 s=10 u+S_{0 a} \quad(\div 10)$
$u+2 a=79$

$$
U+S a=29.5
$$


2. Two particles $P$ and $Q$ are moving in opposite directions along the same horizontal straight line. Particle $P$ has mass $m$ and particle $Q$ has mass km . The particles collide directly. Immediately before the collision, the speed of $P$ is $u$ and the speed of $Q$ is $2 u$. As a result of the collision, the direction of motion of each particle is reversed and the speed of each particle is halved.
(a) Find the value of $k$.
(b) Find, in terms of $m$ and $u$ only, the magnitude of the impulse exerted on $Q$ by $P$ in the collision.

$\qquad$

(a)

$$
\begin{aligned}
m_{1} u_{1}+m_{2} u_{2} & =m_{1} v_{1}+m_{2} v_{2} \\
\psi(u)+K p h(-2 u) & =\phi\left(\frac{1}{2} u\right)+K \mu h(u) \quad(\div m) \\
u-2 K u & =-\frac{1}{2} u+K u \\
u+\frac{1}{2} u & =K u+2 K u \\
\frac{3}{2} u & =3 K u \quad(\div u) \\
3 K & =\frac{3}{2} \quad(\div 3) \\
K & =\frac{1}{2}
\end{aligned}
$$

(b)

$$
\begin{aligned}
& I=m(v-u) \\
& I=\frac{1}{2} m(u-(-2 u)) \\
& I=\frac{1}{2} m(3 u) \\
& I=\frac{3}{2} m u \mathrm{Nm}
\end{aligned}
$$

3. A block $A$ of mass 9 kg is released from rest from a point $P$ which is a height $h$ metres above horizontal soft ground. The block falls and strikes another block $B$ of mass 1.5 kg which is on the ground vertically below $P$. The speed of $A$ immediately before it strikes $B$ is $7 \mathrm{~m} \mathrm{~s}^{-1}$. The blocks are modelled as particles.
(a) Find the value of $h$.

Immediately after the impact the blocks move downwards together with the same speed and both come to rest after sinking a vertical distance of 12 cm into the ground. Assuming that the resistance offered by the ground has constant magnitude $R$ newtons,
(b) find the value of $R$.

(b) $m_{1} u_{1}+m_{2} u_{2}=\left(m_{1}+m_{2}\right) v \quad$ Before Impact After Impact
 10.5 $V=63$ $V=6 \mathrm{~ms}{ }^{-1}$
$V^{2}=u^{2}+2 a s$
$0=b^{2}+2 n(0.12)$
$0.24 a=-36$
$a=-13 \mathrm{oms}^{-2}$


Resultant Force $=\mathrm{ma}$
$10.5 \mathrm{~g}-R=10.5(-150)$
10.5(9.8)-R $=-1575$
$R=1575+102.9$
$R=167.9 \simeq 1680 \mathrm{~N}$

10.59
4.


Figure 1
A diving board $A B$ consists of a wooden plank of length 4 m and mass 30 kg . The plank is held at rest in a horizontal position by two supports at the points $A$ and $C$, where $A C=0.6 \mathrm{~m}$, as shown in Figure 1. The force on the plank at $A$ acts vertically downwards and the force on the plank at $C$ acts vertically upwards.

A diver of mass 50 kg is standing on the board at the end $B$. The diver is modelled as a particle and the plank is modelled as a uniform rod. The plank is in equilibrium.
(a) Find
(i) the magnitude of the force acting on the plank at $A$,
(ii) the magnitude of the force acting on the plank at $C$.

The support at $A$ will break if subjected to a force whose magnitude is greater than 5000 N .
(b) Find, in kg , the greatest integer mass of a diver who can stand on the board at $B$ without breaking the support at $A$.
(c) Explain how you have used the fact that the diver is modelled as a particle.
(a) $\uparrow$

$$
\begin{aligned}
& R_{c}=R_{A}+30 g+\log \\
& R_{c}=R_{A}+80 g
\end{aligned}
$$



Moments Around $A$

$$
\begin{gathered}
R_{c}(0.6)=30 g(2)+\log (4) \\
0.6 R_{c}=60 \mathrm{~g}+200 \mathrm{~g} \\
0.6 R_{c}=260 \mathrm{~g} \quad(\div 0.6) \\
R_{c}=\frac{1300}{3} \simeq 4250 \mathrm{~N}
\end{gathered}
$$

$$
\frac{1300}{3} g=R_{4}+80 g
$$

$$
\begin{aligned}
& R_{A}=-80 \mathrm{~g}+\frac{1300 \mathrm{~g}}{3} \\
& R_{A}=\frac{1060 \mathrm{~g}}{3} \simeq 34+60 \mathrm{~N}
\end{aligned}
$$

Question 4 continued
(b) Moments Around C

$$
\begin{aligned}
& R_{A}<S 000 \\
& 709+\frac{17}{3} \mathrm{mg}<5000 \\
& \begin{aligned}
\frac{12}{3} m g & <5000-70 g \quad\left(\div \frac{17}{3} g\right) \\
m & <77.68
\end{aligned}
\end{aligned}
$$

$\therefore$ Maximum $m$ is 77 Kg
(c) The weight of the diver acts precisely at $B$
$\qquad$
$\qquad$
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5. Two forces, $\mathbf{F}_{1}$ and $\mathbf{F}_{2}$, act on a particle $A$.
$\mathbf{F}_{1}=(2 \mathbf{i}-3 \mathbf{j}) \mathrm{N}$ and $\mathbf{F}_{2}=(p \mathbf{i}+q \mathbf{j}) \mathrm{N}$, where $p$ and $q$ are constants.
Given that the resultant of $\mathbf{F}_{1}$ and $\mathbf{F}_{2}$ is parallel to $(\mathbf{i}+2 \mathbf{j})$,
(a) show that $2 p-q+7=0$

Given that $q=11$ and that the mass of $A$ is 2 kg , and that $\mathbf{F}_{1}$ and $\mathbf{F}_{2}$ are the only forces acting on $A$,
(b) find the magnitude of the acceleration of $A$.
(a)

$$
\begin{aligned}
& R=(2 i-3 j)+(p i+q j) \\
& R=(2+p) i+(q-3) j
\end{aligned}
$$

$$
\begin{aligned}
\because & \text { It is paralk }(\text { to }(i+2 j) \\
\therefore & \frac{q-3}{2+p}=<\frac{2}{1} \\
& 4+2 p=q-3 \\
& 2 p-q+4+3=0 \\
& 2 p-q+7=0 \quad \Perp \angle Q E D
\end{aligned}
$$

(b) $R=(2+p) i+(q-3)$;

$$
\begin{gathered}
2 p-11+7=0 \\
2 p-4=0 \\
p=2 \\
R=(4 i+8 j) N
\end{gathered}
$$

$$
\begin{aligned}
& F=m a \\
& 4 i+8 j=2 a \\
& a=(2 i+4 j) m s^{-2} \\
& |a|=\sqrt{2^{2}+4^{2}}=2 \sqrt{s} \simeq 4.47 m s^{-2}
\end{aligned}
$$

6. 



Figure 2
Two cars, $A$ and $B$, move on parallel straight horizontal tracks. Initially $A$ and $B$ are both at rest with $A$ at the point $P$ and $B$ at the point $Q$, as shown in Figure 2. At time $t=0$ seconds, $A$ starts to move with constant acceleration $a \mathrm{~m} \mathrm{~s}^{-2}$ for 3.5 s , reaching a speed of $14 \mathrm{~m} \mathrm{~s}^{-1}$. Car $A$ then moves with constant speed $14 \mathrm{~m} \mathrm{~s}^{-1}$.
(a) Find the value of $a$.

Car $B$ also starts to move at time $t=0$ seconds, in the same direction as car $A$. Car $B$ moves with a constant acceleration of $3 \mathrm{~m} \mathrm{~s}^{-2}$. At time $t=T$ seconds, $B$ overtakes $A$. At this instant $A$ is moving with constant speed.
(b) On a diagram, sketch, on the same axes, a speed-time graph for the motion of $A$ for the interval $0 \leqslant t \leqslant T$ and a speed-time graph for the motion of $B$ for the interval $0 \leqslant t \leqslant T$.
(c) Find the value of $T$.
(d) Find the distance of car $B$ from the point $Q$ when $B$ overtakes $A$.
(e) On a new diagram, sketch, on the same axes, an acceleration-time graph for the motion of $A$ for the interval $0 \leqslant t \leqslant T$ and an acceleration-time graph for the motion of $B$ for the interval $0 \leqslant t \leqslant T$.

$$
U=0 \quad t=3.5 \quad a=? ? \quad V=14
$$

$V=U_{+a} t$
$1 \angle=0+a(3.5) \quad(\therefore 3.5)$
$a=4 \mathrm{~ms}^{-2}$

(c) Foc $\operatorname{lar} B \quad u=0 \quad a=3$ t=T $v=$ ??

$$
\begin{gathered}
V=U+a t \\
V=0+3 T \\
V=3 T
\end{gathered}
$$

$\because$ The 2 cars travelled the same drstance

$$
\begin{gathered}
\therefore \quad \frac{1}{2}(14)(T+T-3.5)=\frac{1}{2} \times T \times 3 T \quad\left(\div \frac{1}{2}\right) \\
14(2 T-3,5)=3 T^{2} \\
28 T-49=3 T^{2} \\
3 T-28 T+149=0 \\
(3 T-7)(T-7)=0 \\
T=\frac{7}{3} s, T=75
\end{gathered}
$$

Rejected bensee var $A$
uns still acceleating
(d) Distance travelled $=\frac{1}{2} \times 7 \times 21$

$$
=73.5 \mathrm{~m}
$$


7.


Figure 3
A particle $P$ of mass 4 kg is attached to one end of a light inextensible string. A particle $Q$ of mass $m \mathrm{~kg}$ is attached to the other end of the string. The string passes over a small smooth pulley which is fixed at a point on the intersection of two fixed inclined planes. The string lies in a vertical plane that contains a line of greatest slope of each of the two inclined planes. The first plane is inclined to the horizontal at an angle $\alpha$, where $\tan \alpha=\frac{3}{4}$ and the second plane is inclined to the horizontal at an angle $\beta$, where $\tan \beta=\frac{4}{3}$. Particle $P$ is on the first plane and particle $Q$ is on the second plane with the string taut, as shown in Figure 3.

The first plane is rough and the coefficient of friction between $P$ and the plane is $\frac{1}{4}$. The
second plane is smooth. The system is in limiting equilibrium.
Given that $P$ is on the point of slipping down the first plane,
(a) find the value of $m$,
(b) find the magnitude of the force exerted on the pulley by the string,
(c) find the direction of the force exerted on the pulley by the string.

Question 7 continued
(a) Particle $P$


$$
\begin{array}{rl}
\uparrow R & R=4 g \cos \alpha \\
R & =4 g\left(\frac{4}{5}\right) \\
R & =\frac{16}{6} g N \\
\longleftrightarrow & T+M R=4 g \sin \alpha \\
& T+\frac{1}{4}\left(\frac{16}{5} g\right)=4 g\left(\frac{3}{5}\right) \\
& T+\frac{4}{3} g=\frac{12}{5} g \\
& I=\frac{8}{5} g N
\end{array}
$$

$$
\begin{aligned}
\leftrightarrow & =m g \sin \beta \\
\frac{8}{5} g & =m g\left(\frac{4}{5}\right) \\
m & =2 \mathrm{Kg}
\end{aligned} \quad\left(\div \frac{4}{5} g\right)
$$



$$
\tan \alpha=\frac{3^{4}}{4}
$$

$$
\tan \beta=\frac{4}{3}
$$

$$
\cos \alpha=\frac{4}{5}
$$

$$
\sin \alpha=\frac{3}{5}
$$

$$
4 \cos \beta=\frac{3}{4}
$$

Patrice
(b) Resultant $=\sqrt{\left(\frac{8}{5} 9\right)^{2}+\left(\frac{8}{5} 9\right)^{2}}=\frac{8}{5} \sqrt{2} g \simeq 22.2 \mathrm{~N}$
(c) Direction $\rightarrow$ Makes an angle $45^{\circ}$ with the First b- second plane


